

# Offset Sine Guilloche Math

by Bill Ooms

When cutting sine wave patterns on guilloche with a phase shift between successive cuts, it is important to know how to calculate the proper spacing so that a portion of one cut is coincident with a portion of the next cut. This will give a very neat and clean appearance. The formulas below can be easily implemented in a spreadsheet. They apply to both straight line as well as rose engine work.

$\Delta$  = offset between sine wave cuts

$N$  = number of shifted sine waves before the pattern repeats ( $N = 6$  shown below)

$A$  = peak-to-peak amplitude of the sine wave

**Amplitude for a given offset:**

$$A = \Delta \cdot \frac{N}{\pi}$$

**Offset for a given amplitude:**

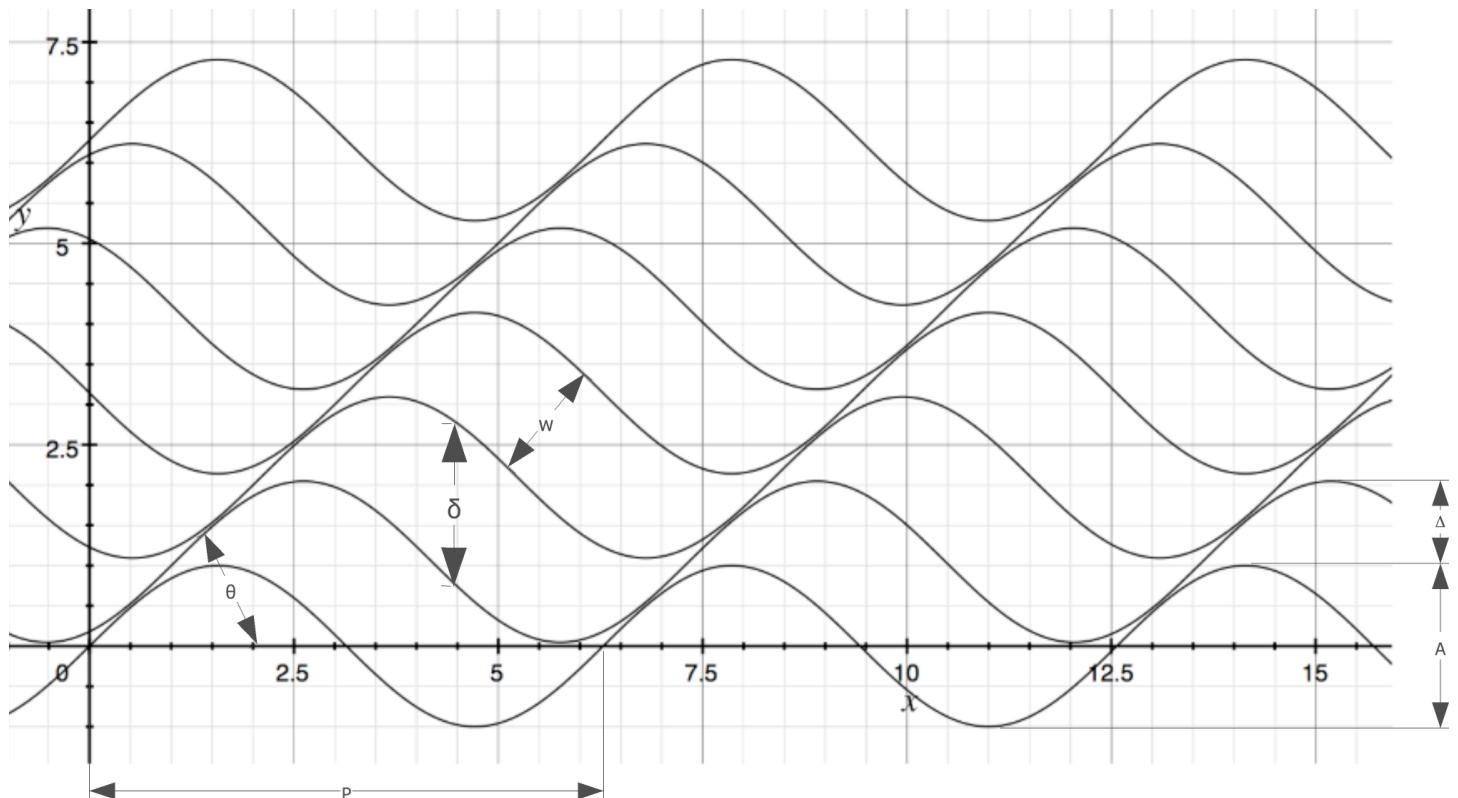
$$\Delta = A \cdot \frac{\pi}{N}$$

The relationship between period of the pattern bar and the angle of the coincident line is given by the following equation:

$P$  = period of the sine wave

$\theta$  = the angle of the coincident line

$$\theta = \text{atan}\left(\frac{A\pi}{P}\right) = \text{atan}\left(\frac{\Delta N}{P}\right)$$



For a fixed cutter (used when cutting metal), we can calculate the required width of cut and the required cut depth. The width required for a rotating single flute cutter (used when making guilloche-like cuts in wood) is different than that of a fixed cutter.

**For a fixed cutter:**

$\alpha$  = the included angle of the cutter  
 $\delta$  = required width of cut for a fixed cutter  
 $d$  = required depth for a fixed cutter

$$\delta = A \cdot \sin\left(\frac{\pi}{N}\right) + \Delta$$

$$d = \frac{\delta}{2 \cdot \tan\left(\frac{\alpha}{2}\right)}$$

**For a rotating cutter:**

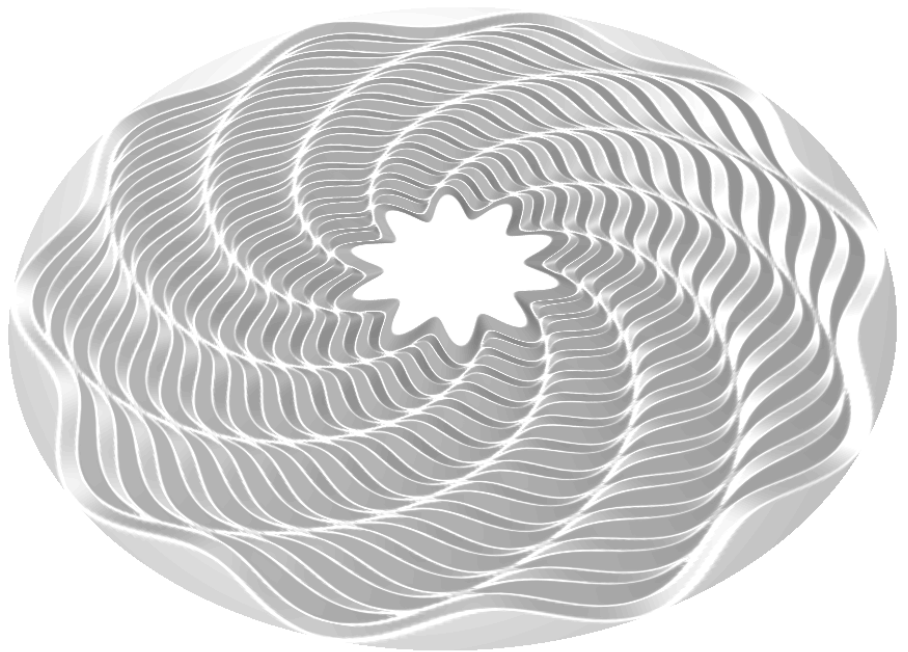
$\alpha$  = the included angle of the cutter  
 $w$  = required width of a rotating cutter  
 $d$  = required depth of a rotating cutter

$$w = \delta \cdot \cos(\theta)$$

$$d = \frac{w}{2 \cdot \tan\left(\frac{\alpha}{2}\right)}$$

**Example:**

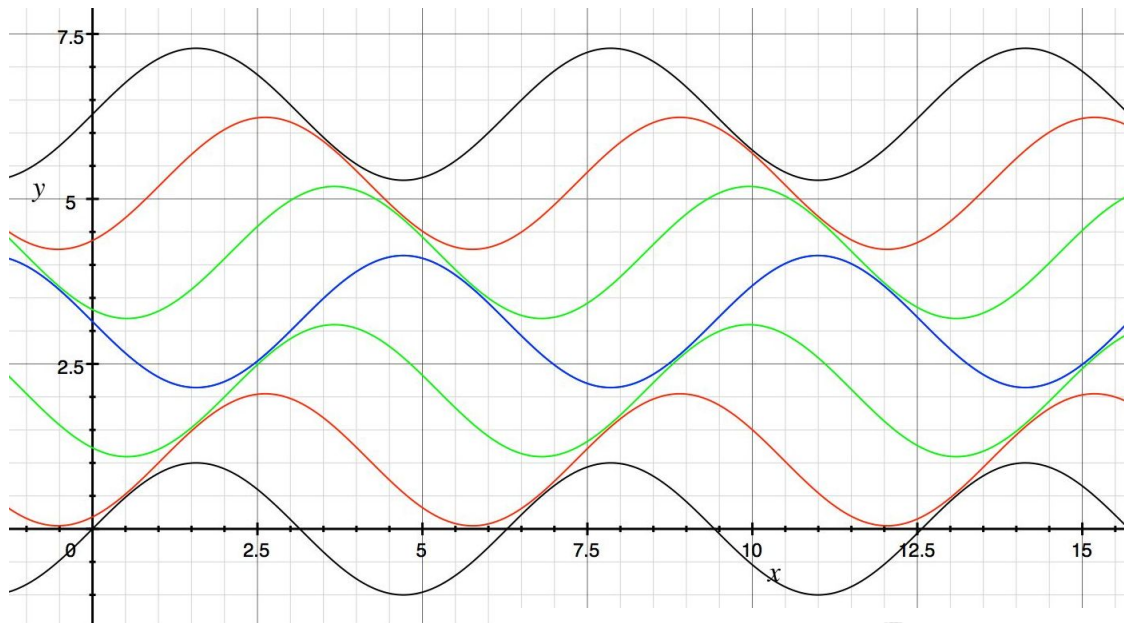
$\Delta = 0.020$ " spacing  
 $N = 6$  phase shifts  
 $A = 0.038$ " amplitude  
 $\alpha = 155^\circ$  fixed cutter  
 $\delta = 0.039$ " width of cut  
 $d = 0.0043$ " cut depth



# Basketweave Guilloche Math

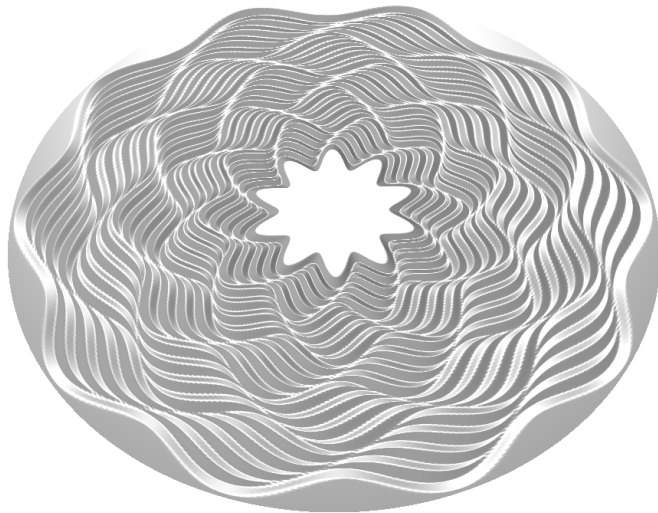
by Bill Ooms

A simple basketweave pattern uses the same equations as for an offset sine pattern. As shown below, you make some number of cuts increasing the phase, then make the same number of cuts decreasing the phase back to the original phase.  $N = 6$  in the image below.



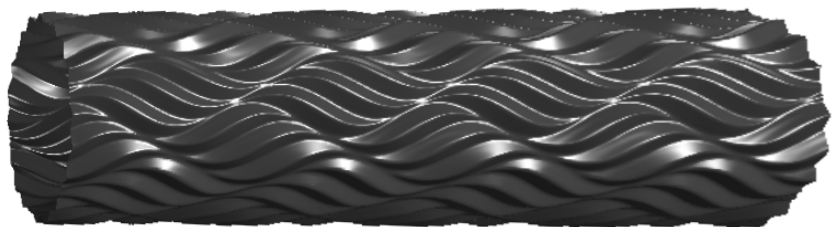
## Example:

$\Delta = 0.015''$  spacing  
 $N = 6$  phase shifts  
 $A = 0.029''$  amplitude  
 $\alpha = 155^\circ$  fixed cutter  
 $\delta = 0.029''$  width of cut  
 $d = 0.0033''$  cut depth



## Example2:

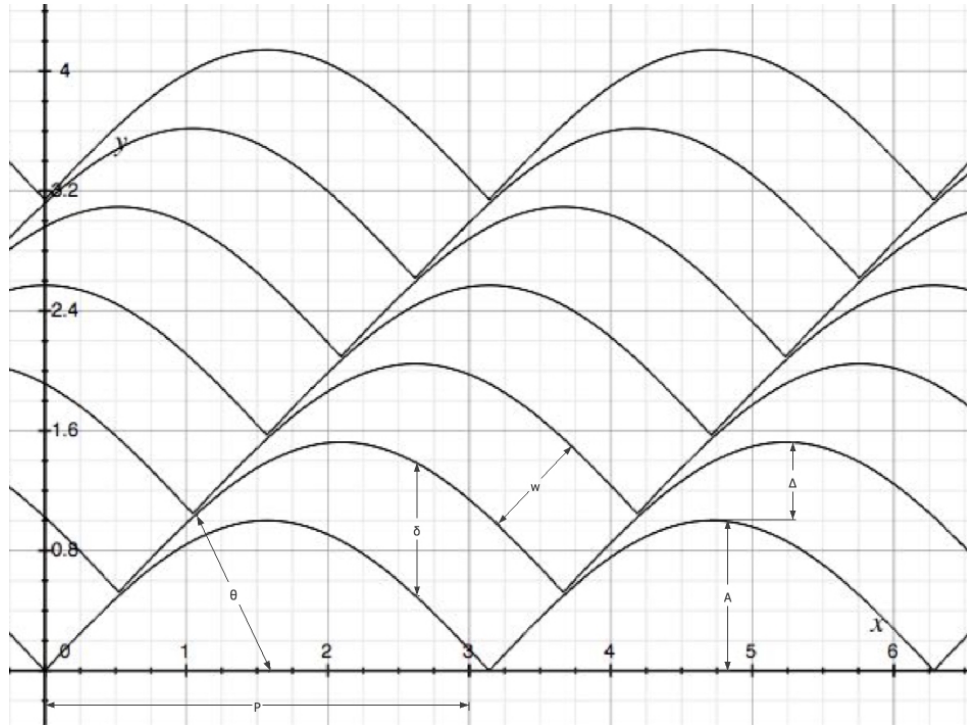
$\Delta = 0.044''$  spacing  
 $N = 6$  phase shifts  
 $A = 0.085''$  amplitude  
 $P = 0.5''$  period of sine  
 $\theta = 30^\circ$  angle of line  
 $\alpha = 120^\circ$  rotating cutter  
 $w = 0.076''$  width of cut  
 $d = 0.022''$  cut depth



# Offset Half Sine Math

by Bill Ooms

Offsetting a half sine wave pattern uses equations that are essentially the same as those for an offset sine pattern.  $N = 6$  in the image below.



## Example:

$\Delta = 0.020''$  spacing  
 $N = 12$  phase shifts  
 $A = 0.076''$  amplitude  
 $\alpha = 150^\circ$  fixed cutter  
 $\delta = 0.040''$  width of cut  
 $d = 0.0044''$  cut depth

